

# MATHEMATICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1.  $\cot^{-1}\left(\frac{7}{4}\right) + \cot^{-1}\left(\frac{19}{4}\right) + \cot^{-1}\left(\frac{39}{4}\right) + \dots \infty$

(1)  $\cot^{-1}(2)$                       (2)  $\cot^{-1}\left(\frac{1}{2}\right)$

(3)  $\cot^{-1}\left(\frac{1}{3}\right)$                       (4)  $\cot^{-1}(3)$

**Answer (2)**

**Sol.**  $T_r = \cot^{-1}\left(\frac{4r^2 + 3}{4}\right)$

$$T_r = \tan^{-1}\left(\frac{1}{r^2 + \frac{3}{4}}\right)$$

$$T_r = \tan^{-1}\left(\frac{\left(r + \frac{1}{2}\right) - \left(r - \frac{1}{2}\right)}{1 + r^2 - \frac{1}{4}}\right)$$

$$T_r = \tan^{-1}\left(\frac{\left(r + \frac{1}{2}\right) - \left(r - \frac{1}{2}\right)}{1 + \left(r + \frac{1}{2}\right)\left(r - \frac{1}{2}\right)}\right)$$

$$T_r = \tan^{-1}\left(r + \frac{1}{2}\right) - \tan^{-1}\left(r - \frac{1}{2}\right)$$

$$T_1 = \tan^{-1}\left(\frac{3}{2}\right) - \tan^{-1}\left(\frac{1}{2}\right)$$

$$T_2 = \tan^{-1}\left(\frac{5}{2}\right) - \tan^{-1}\left(\frac{3}{2}\right)$$

$$T_n = \tan^{-1}\left(\frac{2n+1}{2}\right) - \tan^{-1}\left(\frac{1}{2}\right)$$

$$\sum T_r = \tan^{-1}\left(\frac{2n+1}{2}\right) - \tan^{-1}\left(\frac{1}{2}\right)$$

$$\sum T_r = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)$$

$$\sum T_r = \cot^{-1}\left(\frac{1}{2}\right)$$

2.  $\int \frac{(\sqrt{1+x^2} + x)^{19}}{\sqrt{1+x^2} - x} dx$  is equal to

(1)  $\frac{1}{21}(\sqrt{1+x^2} + x)^{20} + \frac{1}{19}(\sqrt{1+x^2} + x)^{19} + c$

(2)  $\frac{1}{36}(\sqrt{1+x^2} + x)^{18} + \frac{1}{38}(\sqrt{1+x^2} + x)^{19} + c$

(3)  $\frac{1}{42}(\sqrt{1+x^2} + x)^{21} + \frac{1}{38}(\sqrt{1+x^2} + x)^{19} + c$

(4)  $\frac{1}{21}(\sqrt{1+x^2} + x)^{21} + \frac{1}{19}(\sqrt{1+x^2} + x)^{20} + c$

**Answer (3)**

**Sol.**  $I = \int (\sqrt{1+x^2} + x)^{20} dx$

Let  $x = \tan\theta$

$\therefore dx = \sec^2\theta d\theta$

$$I = \int (\sec + \tan\theta)^{20} \cdot \sec^2\theta d\theta$$

$$= \int \sec\theta (\sec\theta + \tan\theta)^{20} \cdot \frac{(\sec\theta + \tan\theta) + (\sec\theta - \tan\theta)}{2} d\theta$$

$$= \frac{1}{2} \int \sec\theta (\sec\theta + \tan\theta)(\sec\theta + \tan\theta)^{20} d\theta$$

$$+ \frac{1}{2} \int \sec\theta (\sec + \tan\theta)^{20} (\sec\theta - \tan\theta) d\theta$$

$$= \frac{1}{2} \int \sec\theta (\sec\theta + \tan\theta)(\sec\theta + \tan\theta)^{20} d\theta$$

$$+\frac{1}{2}\int \sec\theta \cdot (\sec\theta + \tan\theta) \cdot (\sec\theta + \tan\theta)^{18} d\theta$$

Let  $\sec\theta + \tan\theta = u$

$$\Rightarrow \sec\theta(\sec\theta + \tan\theta) = \frac{du}{d\theta}$$

$$= \frac{1}{2}\int u^{20} du + \frac{1}{2}\int u^{18} d\theta$$

$$= \frac{u^2}{42} + \frac{u^{19}}{38} + c$$

$$= \frac{1}{42}(\sec\theta + \tan\theta)^{21} + \frac{1}{38}(\sec\theta + \tan\theta)^{19} + c$$

$$= \frac{1}{42}(\sqrt{1+x^2} + x)^{21} + \frac{1}{38}(\sqrt{1+x^2} + x)^{19} + c$$

3. Let  $L_1: \frac{x-1}{3} = \frac{y}{4} = \frac{z}{5}$  and  $L_2: \frac{x-p}{2} = \frac{y}{3} = \frac{z}{4}$ . If the shortest distance between  $L_1$  and  $L_2$  is  $\frac{1}{\sqrt{6}}$ . Then possible value of  $p$  is

- (1) 3  
(2) 2  
(3) 5  
(4) 7

**Answer (2)**

**Sol.** S.D. between two given lines  $L_1$  and  $L_2$

$$= \frac{\begin{vmatrix} p-1 & 0 & 0 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}}{|(2\hat{i} + 3\hat{j} + 4\hat{k}) \times (3\hat{i} + 4\hat{j} + 5\hat{k})|}$$

$$= \frac{|(p-1)(-1)|}{\sqrt{6}}$$

$$\therefore \frac{|p-1|}{\sqrt{6}} = \frac{1}{\sqrt{6}}$$

$$\therefore p-1 = \pm 1$$

$$\therefore p = 0 \text{ or } 2$$

4. Let  $f(x)$  and  $g(x)$  satisfies the functional equation  $2g(x) + 3g\left(\frac{1}{x}\right) = x$  and  $2f(x) + 3f\left(\frac{1}{x}\right) = x^2 + 5$ .

If  $\alpha = \int_1^2 f(x) dx$  and  $\beta = \int_1^2 g(x) dx$  then  $(9\alpha + \beta)$  is equal to

(1)  $\frac{27+6\ln_2}{10}$

(2)  $\frac{27-6\ln_2}{10}$

(3)  $\frac{3}{5}\ln_2$

(4)  $\frac{3}{5}\ln_2 + \frac{7}{30}$

**Answer (1)**

**Sol.**  $2g(x) + 3g\left(\frac{1}{x}\right) = x \Rightarrow 6g(x) + 9g\left(\frac{1}{x}\right) = 3x$

$$2g\left(\frac{1}{x}\right) + 3g(x) = \frac{1}{x} \Rightarrow 6g(x) + 4g\left(\frac{1}{x}\right) = \frac{2}{x}$$

$$\Rightarrow 5g\left(\frac{1}{x}\right) = 3x - \frac{2}{x}$$

$$\Rightarrow g(x) = \frac{1}{5}\left(\frac{3}{x} - 2x\right)$$

Similarly,

$$2f(x) + 3f\left(\frac{1}{x}\right) = x^2 + 5 \Rightarrow 4f(x) + 6f\left(\frac{1}{x}\right) = 2x^2 + 10$$

$$3f(x) + 2f\left(\frac{1}{x}\right) = 5 + \frac{1}{x^2} \quad 9f(x) + 6f\left(\frac{1}{x}\right) = \frac{15+3}{x^2}$$

$$\Rightarrow 5f(x) = \frac{15+3}{x^2} - 2x^2 - 10$$

$$\Rightarrow f(x) = \frac{1}{5}\left[\frac{5+3}{x^2} - 2x^2\right]$$

$$\alpha = \int_1^2 f(x) dx = \frac{11}{30^2} \quad \text{and} \quad \beta = \int_1^2 g(x) dx = \frac{3}{5}\ln x - \frac{x^2}{5}\Big|_1^2$$

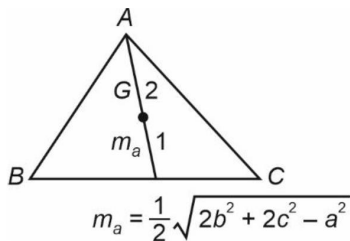
$$= \left(\frac{3}{5}\ln_2 - \frac{4}{5}\right) - \left(\frac{-1}{5}\right) = \frac{3}{5}(\ln_2 - 1)$$

5. In  $\triangle ABC$  if  $A$  is  $(2, -3, 1)$ ,  $B$  is  $(3, -1, -5)$  and  $C$  is  $(1, -4, -4)$ . If  $G$  is the centroid of  $\triangle ABC$ , then  $6((AG)^2 + (BG)^2 + (CG)^2)$  is equal to

- (1) 164 (2) 265  
(3) 625 (4) 628

**Answer (1)**

**Sol.**



$$AB = \sqrt{1^2 + 2^2 + 6^2} = \sqrt{41}$$

$$BC = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$$

$$AC = \sqrt{1^2 + 1^2 + 5^2} = \sqrt{27}$$

$$AG^2 + BG^2 + CG^2 = \frac{1}{3}(AB^2 + BC^2 + AC^2)$$

$$= \frac{1}{3}(41 + 14 + 27)$$

$$6(AG^2 + BG^2 + CG^2) = 2(82)$$

$$= 164$$

6. If  $m$  and  $n$  are two digit numbers such that  $m > n$  and  $\gcd(m, n) = 6$ . Find the number of such pairs.

- (1) 60 (2) 65  
(3) 55 (4) 35

**Answer (2)**

**Sol.**  $m, n$  two digit number

$$m > n$$

$$\gcd(m, n) = 6$$

$$\Rightarrow m = 6a$$

$$n = 6b \text{ such that } \gcd(a, b) = 1, a > b$$

$$\text{Now, since } n \geq 10 \Rightarrow b \geq 2$$

$$m \leq 99 \Rightarrow a \leq 16$$

We need to find  $a, b$  such that

$$\gcd(a, b) = 1, a > b \text{ and } a, b \in \{2, 3, \dots, 16\}$$

Let  $b = 2$ ,  $a$  will be all odd numbers

$$a \in \{3, 5, 7, 9, 11, 13, 15\} \Rightarrow 7 \text{ pairs}$$

Let  $b = 3$ ,  $a \in \{4, 5, 7, 8, 10, 11, 13, 14, 16\} \Rightarrow 9$  pairs

Let  $b = 4$ ,  $a \in \{5, 7, 9, 11, 13, 15\} \Rightarrow 6$  pairs

Let  $b = 5$ ,  $a \in \{6, 7, 8, 9, 11, 12, 13, 14, 16\} \Rightarrow 9$  pairs

Let  $b = 6$ ,  $a \in \{7, 11, 13\} \Rightarrow 3$  pairs

Let  $b = 7$ ,  $a \in \{8, 9, 10, 11, 12, 13, 14, 15, 16\} \Rightarrow 9$  pairs

Let  $b = 8$ ,  $a \in \{9, 11, 13, 15\} \Rightarrow 4$  pairs

Let  $b = 9$ ,  $a \in \{10, 11, 13, 14, 16\} \Rightarrow 5$  pairs

Let  $b = 10$ ,  $a \in \{11, 13\} \Rightarrow 2$  pairs

Let  $b = 11$ ,  $a \in \{12, 13, 14, 15, 16\} \Rightarrow 5$  pairs

Let  $b = 12$ ,  $a \in \{13\} \Rightarrow 1$  pair

Let  $b = 13$ ,  $a \in \{14, 15, 16\} \Rightarrow 3$  pairs

Let  $b = 14$ ,  $a \in \{15\} \Rightarrow 1$  pair

Let  $b = 15$ ,  $a \in \{16\} \Rightarrow 1$  pair

Total  $\Rightarrow 65$  numbers

7. The sum of three terms in A.P. "A" is 36 and product is  $p$  and for A.P. "B" sum is 36 and product is  $q$  of three terms and  $\frac{p+q}{p-q} = \frac{19}{5}$ , given  $D = d + 3$ . Then the value of  $p-q$ . (where  $d$  and  $D$  are common difference of A.P. "A" and "B", respectively)

(1) 620 (2) 540

(3) 510 (4) 720

**Answer (1)**

**Sol.** Let the terms to  $a_1 - d, a_1, a_1 + d$ , and

$$a_2 - D, a_2, a_2 + D$$

$$a_1 - d + a_1 + a_1 + d = 36 \Rightarrow a_1 = 12$$

$$a_1 = a_2 = 12$$

$$(a_1 - d)(a_1)(a_1 + d) = p$$

$$\frac{p+q}{p-q} = \frac{19}{5} \Rightarrow 5p + 5q = 19p - 19q$$

$$\Rightarrow 24q = 14p \Rightarrow 12q = 7p$$

$$12(12 - D)(12)(12 + D) = 7(12 - d)12(12 + d)$$

$$\Rightarrow 12(9 - d)(15 + d) = 7(12 - d)(12 + d)$$

$$= 12(135 - 6d - d^2) = 7(144 - d^2)$$

$$= 5d^2 + 72d - 612 = 0 \Rightarrow (5d - 102)(d - 6) = 0$$

$$\Rightarrow d = 6, D = 9$$

$$P = (12 - 6)(12)(12 + 6) = 6 \times 12 \times 18$$

$$q = (12 - 9)(12)(12 + 9) = 3 \times 12 \times 21$$

$$p - q = 12 \times 9(12 - 7) = 108 \times 5 = 540$$

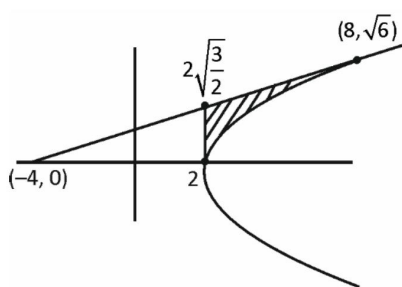
8. The area bounded between the tangent of a parabola  $y^2 = x - 2$  which passes through  $(-4, 0)$  with positive slope,  $x$ -axis, and the parabola is (in square units)

(1)  $\frac{7\sqrt{6}}{2}$  (2)  $\sqrt{6}$

(3)  $\frac{3\sqrt{6}}{2}$  (4)  $\frac{9}{2}\sqrt{6}$

**Answer (4)**

**Sol.**



$$y = (x - 2)m + \frac{1}{4m}$$

using  $y = mx + \frac{a}{m}$  for  $y^2 = x - 2$

$$\Rightarrow m = \pm \frac{1}{\sqrt{24}}$$

$$\Rightarrow \text{line is } \sqrt{24}y = x + 4$$

$$\text{Area of triangle} = \frac{1}{2} \times 6 \times \sqrt{\frac{3}{2}} = 3\sqrt{\frac{3}{2}} = \sqrt{\frac{27}{2}} = 3\sqrt{\frac{3}{2}}$$

$$= \frac{3}{2}\sqrt{6}$$

$$\text{Other area} = \int_2^8 \frac{x+4}{\sqrt{24}} - \sqrt{x-2}$$

$$\frac{x^2}{\sqrt{24}} + \frac{4x}{\sqrt{24}} - \frac{(x-2)^{3/2}}{\frac{3}{2}} \Big|_2^8 = \left( \frac{64+32}{\sqrt{24}} - \frac{6\sqrt{6}}{\frac{3}{2}} \right) - \left( \frac{4+8}{\sqrt{24}} - 0 \right)$$

$$= \frac{96-4\sqrt{6}}{\sqrt{24}} - \frac{12}{\sqrt{24}} = \frac{84-4\sqrt{6}}{2\sqrt{6}}$$

$$= \frac{84-48}{2\sqrt{6}} = \frac{36}{2\sqrt{6}} = 3\sqrt{6}$$

9. Let  $f(x) = \log_2(\log_3(\log_7(8 - \log_2(x^2 + 5x + 6))))$  where  $(x < -3)$  has domain  $(\alpha, \beta)$  and

$$g(x) = \sin^{-1}\left(\frac{2x^2 + x + 1}{3x + 5}\right)$$
 has domain  $[\gamma, \delta]$ . Then the

value of  $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$  equals to

(1) 20 (2) 19

(3) 18 (4) 22

**Answer (4)**

**Sol.**  $\log_2(\log_3(\log_7(8 - \log_2(x^2 + 5x + 6))))$

$$\log_3(\log_7(8 - \log_2(x^2 + 5x + 6))) > 0$$

$$\log_2(8 - \log_2(x^2 + 5x + 6)) > 1$$

$$8 - \log_2(x^2 + 5x + 6) > 7$$

$$-\log_2(x^2 + 5x + 6) > -1$$

$$\log_2(x^2 + 5x + 6) < 1$$

$$x^2 + 5x + 6 < 2$$

$$x^2 + 5x + 4 < 0$$

$$(x + 4)(x + 1) < 0 \quad \dots(1)$$

$$\log_7(8 - \log_2(x^2 + 5x + 6)) > 0$$

$$8 - \log_2(x^2 + 5x + 6) > 1$$

$$7 > \log_2(x^2 + 5x + 6)$$

$$2^7 > x^2 + 5x + 6$$

$$-\log_2(x^2 + 5x + 6) > -1$$

$$\Rightarrow x^2 + 5x + 6 < 128$$

$$\Rightarrow x = \frac{-5 \pm \sqrt{25 + 513}}{2}$$

$$x = \frac{-5 \pm \sqrt{513}}{2} \quad \dots(2)$$

$$8 - \log_2(x^2 + 5x + 6) > 0$$

$$\log_2(x^2 + 5x + 6) < 8$$

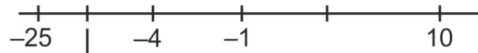
$$x^2 + 5x + 6 < 2^8$$

$$x^2 + 5x + 6 < 256$$

$$\Rightarrow x^2 + 5x - 250 < 0$$

$$\Rightarrow x^2 + 25x - 10x - 250 < 0 \dots(3)$$

Intersection of (1) and (2) and (3) is



$$\frac{-5 - \sqrt{513}}{2} \qquad \frac{-5 - \sqrt{513}}{2}$$

$$\frac{-5 - \sqrt{5,3}}{2}$$

$$x \in (-4, 3)$$

$$\text{Also } 1 \leq \frac{2x^2 + x + 1}{3x + 5} \leq 1$$

$$\Rightarrow x \in [-1, 2]$$

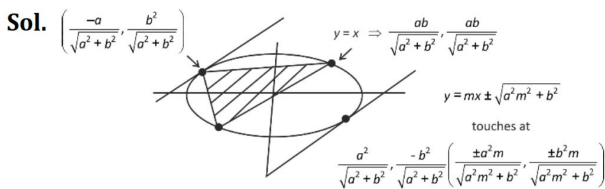
$$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 16 + 9 + 1 + 4 \Rightarrow 30$$

10. Ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  the line  $y = x + p$  is a tangent on it

and touches at points  $P$  and  $Q$ . Also if the line  $y = x$  intersects ellipse at  $R$  and  $S$ . Then area of quadrilateral  $PQRS$  is (Where  $a = 3$  and  $b = 4$ )

- (1) 12                      (2) 10  
(3) 24                      (4) 20

**Answer (3)**



$$\Rightarrow \text{For } m = 1$$

$$A_1 = \frac{1}{2} \begin{vmatrix} \frac{ab}{\sqrt{a^2+b^2}} & \frac{ab}{\sqrt{a^2+b^2}} & 1 \\ \frac{-ab}{\sqrt{a^2+b^2}} & \frac{-ab}{\sqrt{a^2+b^2}} & 1 \\ \frac{-a^2}{\sqrt{a^2+b^2}} & \frac{b^2}{\sqrt{a^2+b^2}} & 1 \end{vmatrix}$$

$$C_1 \rightarrow C_1 + C_2$$

$$A_1 = \frac{1}{2} \begin{vmatrix} 0 & 0 & 2 \\ \frac{-ab}{\sqrt{a^2+b^2}} & \frac{-ab}{\sqrt{a^2+b^2}} & 1 \\ \frac{-a^2}{\sqrt{a^2+b^2}} & \frac{b^2}{\sqrt{a^2+b^2}} & 1 \end{vmatrix}$$

$$A_1 = \frac{1}{2} \left( 2 \cdot \frac{ab^3 + a^3b}{(a^2+b^2)} \right) = \frac{ab(a^2+b^2)}{(a^2+b^2)} = ab$$

Similarly,  $A_2 = ab$

Total area of quadrilateral is  $2ab$  for  $a = 4, b = 3$

$$\text{Area} = 2(4)(3) = 24$$

11. Given data 2, 3, 3, 4, 7, 5,  $a, b$  where mean is 4 and S.D is 1.5. Find the mean deviation about mode.

- (1)  $\frac{19}{8}$                       (2)  $\frac{9}{8}$   
(3)  $\frac{11}{8}$                       (4)  $\frac{5}{8}$

**Answer (2)**

**Sol.**  $\frac{2+3+3+4+7+5+9}{8} = 4$

$$\Rightarrow a + b = 8$$

$$(1.5)^2 = \frac{2+3+3+4+7+5+9^2+6^2}{8} - (4)^2$$

$$\Rightarrow a^2 + b^2 = 34$$

$$\therefore a = 3, b = 5$$

$$\therefore \text{Mode} = 3$$

$$\text{Mean deviation about mode} = \frac{1+1+3+2+2}{8}$$

$$= \frac{9}{8}$$

12.



13.  
14.  
15.  
16.  
17.  
18.  
19.  
20.

**SECTION - B**

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If  $\alpha$  be the non real root of the equation  $1 + x + x^2 = 0$ .

Find  $n$  if  $\sum_{k=1}^n \left( \alpha^k + \frac{1}{\alpha^k} \right)^2 = 20$ .

**Answer (11)**

**Sol.**  $\alpha$  is root of equation  $1 + x + x^2 = 0$ ,  $\alpha = \omega$  or  $\omega^2$

$$\left( \alpha^k + \frac{1}{\alpha^k} \right)^2 = \alpha^{2k} + \frac{1}{\alpha^{2k}} + 2 = (\omega^k)^2 + \left( \frac{1}{\omega^k} \right)^2 + 2$$

$$\Rightarrow \omega^k + \frac{1}{\omega^k} + 2 = \begin{cases} 4, & 3 \text{ divides } k \\ 1, & 3 \text{ doesn't divide } k \end{cases}$$

$$\Rightarrow \sum_{k=1}^n \left( \alpha^k + \frac{1}{\alpha^k} \right)^2 = 20$$

$$(1 + 1 + 4) + (1 + 1 + 4) + (1 + 1 + 4) + (1 + 1) = 20$$

$$\Rightarrow n = 11$$

22. Let  $A = \{-3, -2, -1, 0, 1, 2, 3\}$  and  $xRy \Rightarrow 2x - y \in \{0, 1\}$ . If  $l$  is number of elements in given relation,  $m$  and  $n$  are minimum number of elements to be added to make it reflexive and symmetric respectively. Then  $l + m + n$  equals to

**Answer (17)**

**Sol.**  $2x - y = 0$

$$\Rightarrow 2x = y$$

$$\{(-1, -2), (0, 0), (1, 2)\}$$

$$2x - y = 1$$

$$\Rightarrow 2x = 1 + y$$

$$\{(-1, -3), (0, -1), (1, 1), (2, 4)\}$$

$$\therefore R = \{(-1, -2), (-1, -3), (0, 0), (1, 2), (0, -1),$$

$$(1, 1), (2, 4)\}$$

$$\therefore l = 7$$

$$m = 7 - 2 = 5 \equiv \{(-3, -3), (-2, -2), (-1, -1), (2, 2), (3, 3)\}$$

$$n = 5 \equiv \{(-2, -1), (-3, -1), (2, 1), (-1, 0), (4, 2)\}$$

$$\therefore l + m + n = 17$$

23. If  $1^{2 \cdot 15} C_1 + 2^{2 \cdot 15} C_2 + \dots + 15^{2 \cdot 15} C_{15}$  is equal to  $2^n \cdot 3^m \cdot 5^k$ , then  $m + n + k$  is equal to

**Answer (19)**

**Sol.**  $\sum_{r=1}^{15} r^{215} C_r \qquad (r^n C_r = n^{n-1} C_{r-1})$

$$= 15 \sum_{r=1}^{15} r \cdot 14 C_{r-1}$$

$$= 15 \sum_{r=1}^{15} (r-1+1) 14 C_{r-1}$$

$$= 15 \sum_{r=1}^{15} (r-1) 14 C_{r-1} + 15 \sum_{r=1}^{15} 14 C_{r-1}$$

$$15 \cdot \sum_{r=0}^{14} r 14 C_r + 15 \cdot (2^{14})$$

$$= 15 \times 14 \sum_{r=0}^{14} 13 C_{r-1} + 15 \cdot 2^{14}$$

$$= 15 \times 14 \cdot 2^{13} + 15 \cdot 2^{14}$$

$$= 2^{14} (15 \times 7 + 15)$$

$$= 2^{14} \times 15(8)$$

$$= 2^{17} \cdot 3 \cdot 5$$

$$\therefore n + m + k = 19$$

24. If  $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$  and  $A^n = A^{n-1} + A^2 - I \forall n \geq 3$ , then sum

of all elements of  $A^{50}$  is

**Answer (53.00)**

**Sol.** If  $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^5 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}, A^6 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}, \text{ similarly}$$

$$A^5 = \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \text{Sum of elements} = 53$$

25.